Quiz 3 Polymer Physics Fall 2000
10/11/00

The primary response function is defined by the equation:

$$x(t) = \int_{-\infty}^{t} \mu(t - t')\psi(t')dt'$$

a) **Why** is the argument of the primary response function (t - t') rather than (t) or (t')?

b) **-Sketch** the primary response function for a Newtonian fluid and for a simple relaxatory system as a function of time.
Consider a stress Ψ(t) that is composed of a series of equal pulses in time.
-**Which** of these two systems (Newtonian or relaxatory) weight more recent stresses higher in terms of the resulting strain?
-**Explain** your answer.

c) The following equations defines the dynamic susceptibility as the Fourier transform of the primary response function.

$$\alpha^*(\omega) = \int_{-\infty}^{t} \mu(t - t')\exp\{i\omega(t - t')\}dt' = \int_{0}^{\infty} \mu(t'')\exp\{-i\omega t''\}dt'' = \int_{-\infty}^{\infty} \mu(t'')\exp\{-i\omega t''\}dt''$$

-Show how the first integral can be converted to the second integral.
-What feature of the primary response function allows the final equality to be made?

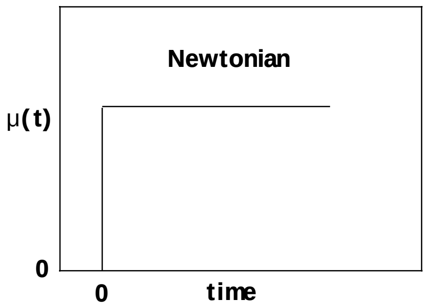
d) The Fourier transform in part "c" converts from the time domain, t, to the frequency domain, ω. Consider a perfect harmonic oscillator (not damped).
-Sketch the primary response function versus time for this perfect oscillator.
-Explain how this signal can be decomposed (Fourier transformed) into responses at different frequencies.
-From your explanation sketch the dynamic susceptibility versus frequency for this oscillator.

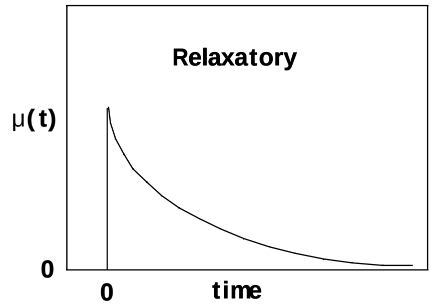
e) The time dependent susceptibility, α(t) has a simple relationship with the primary response function, μ(t).
-Give this relationship.
-For a stress relaxation measurement ($x(t) = x_0$) use the definition of the primary response function (top equation), and the expression for μ(t) in terms of α(t) to obtain an integral relationship between the time dependent modulus, $a(t) = \Psi(t)/x_0$, and α(t).
-How does this compare with the relationship between the static modulus, E, and the static compliance, D?

Answers Quiz 3 Polymer Physics

a) The primary response function acts to weight the applied stress as a function of the time at which it was applied, t', relative to the present time, t. Because of this it is the difference between the present time, t, and the time of application of the stress, t', that is the argument of the function.

b)





-The Relaxatory system will weight more recent stresses higher relative to older stresses. The Newtonian system weights all stresses equally.

c) t'' = t - t', so dt' = -dt''. At t' = -∞ (lower limit), t'' = ∞ and at t' = t, t'' = 0 (upper limit). The negative sign flips the integration limits leading to the equality.

The final equality can be made since the primary response function is always equal to 0 before time t = 0. This is a consequence of the assumption of causality.

d) The primary response function is an oscillating sine wave.

Fourier transformation involves decomposing the time domain signal into a series of sine waves of different amplitudes. The signal from the perfect oscillator has only one Fourier component, i.e. has only one frequency associated with it, so results in a delta function in frequency space (a single value in the frequency domain and the resonance frequency, w_0).

e) μ(t) = dα(t)/dt

For stress relaxation x(t) = x_0, so

$$1 = \int_0^t \mu(t-t')a(t')dt' = \int_0^t \frac{d\alpha}{dt}(t-t')a(t')dt'$$

The equivalent expression for static measurements is 1 = E D.